# Pay it Forward

*Lesson Plan for Grade 8, Pre-Algebra*
*Prepared by Mrs. Gehret*

## EDUCATION STANDARDS

Math:

a. 8.EE.A.1: Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, 32 × 3–5 = 3–3 = 1/33 = 1/27
b. 8.EE.A.2: Use square root and cube root symbols to represent solutions to equations of the form $x2 = p$ and $x3 = p$, where $p$ is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that √2 is irrational.
c. 8.F.A.1: Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output
d. 8.F.A.2: Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change*.
e. 8.F.A.3: Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function A = s2giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line*.
f. 8.F.B.5: Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.
g. MP3: Reason abstractly and quantitatively
h. MP4: Model with mathematics

Visual Arts: VA:Re.7.2.8a Compare and contrast contexts and media in which viewers encounter images that influence ideas, emotions, and actions.

## OBJECTIVES

1. SWBAT: model exponential growth and contrast it to linear growth.

## MATERIALS NEEDED

1. Portraits of Peacemakers
2. “Pay it Forward” trailer
3. Math Notebook
4. Writing Utensil
5. Chart Paper
6. Graph Paper
7. Markers
8. Rulers
9. Google Sheets
10. Desmos
11. Graphing Calculators

## ROOM SET-UP REQUIRED

<table>
<tr><td rowspan="2"></td><td></td><td></td><td></td><td></td><td></td><td rowspan="2"></td></tr>
<tr><td colspan="5" rowspan="3"></td></tr>
<tr><td>Empty Space</td><td>Empty Space</td></tr>
<tr><td rowspan="2"></td><td rowspan="2"></td></tr>
<tr><td></td><td></td><td></td><td></td><td></td></tr>
</table>

## LESSON TIMING

# DAY 1

| | |
|---|---|
| Introduction: Reading Portraits Review | 15 minutes |
| Learning Activity: Pay it Forward | 30 minutes |
| Closure: Exit Ticket | 5minutes |

| | |
|---|---|
| Total | 50 minutes |

## INTRODUCTION

### WHAT

*Today, we are going to look at illustrations* (point to your eyes), *think about illustrations* (point to the top of your head), *and talk about illustrations* (show a talking gestures beside your mouth).

### WHY: STATE INSTRUCTIONAL OBJECTIVE

*We are going to look, think, and talk about pictures of peacemakers to help us better understand how to model exponential growth and contrast it to linear growth.*

### HOW

*These pictures are portraits, so remember we need to notice the facial expression, focal point, gesture, and clothing, setting (if there is one!) and the quantity, size, and color of the objects.*

*I am going to place the pictures in the middle of the circle. Your job is to stay flat on your bottom, not to touch, use your eyes to look and your brains to think how these portraits are the same and different. We will look and think about the pictures for one minute by ourselves, talk with a partner for a minute, then we will talk as a whole group. It is very important the first minute is silent so your eyes can look and your brain can think in order to prepare for your conversation.*

### RESTATE INSTRUCTIONAL OBJECTIVE TO TRANSITION INTO LESSON

*It is important we first look and think by ourselves so we can prepare our thoughts for the conversation about how these individuals helped spread peace around the world. What do you see is the same? What do you see that is different? Remember to support your thinking with evidence from the pictures, be prepared to answer the question "what do you see that makes you say that?"*

## LEARNING ACTIVITY

| **TITLE**: |
|---|
| **Step #1**: OBSERVE/COMPARE/CONTRAST<br>Peacemaker images are placed in the middle of the circle or projected so all students can easily view.<br><br>Keep pace engaging as student first think independently (approximately one minute), then talk with a partner (approximately one minute), then facilitate a whole group discussion (approximately 2 minutes). Listen carefully to students' conversations, record on white board what you hear. |

Dig deeper: Ask "what else do you see? What do you see that makes you say that?" When students have significant observations of things that are the same and different, shift their observations to inferences. (e.g., *you notice that many of the artists used dark colors. Why do you think so many artists would use dark colors? What does the setting say about the time period?*)

Write relevant inferences/information that pertain to how these individuals spread peace around the world. It is NOT necessary to write how the images are the same/different, rather write the inferences drawn based on student observations.

**Step #2:** THINK: PREDICT/INFER and INFER/INQUIRE
Organize students in groups of 2 back at their tables. Tell students they will be viewing 1 work of art while playing "Pass the Portrait."

*When you hear a clap and a chant, "pass the por-trait," it means it is time to pass the painting to the left. This group will pass their painting to this group* (point or show them which way to pass). *Pass the painting carefully and quickly and take 10 seconds to answer "how did these individuals spread peace around the world?"*

*Take 10 seconds to decide where you are going to put the painting so both of you can see it. Once I see you have a plan, then I will bring you and your partner your first painting to begin talking about.*

Play "Pass the Portrait" until students have seen approximately five or six works of art. As students are talking listen for the following:

"Are students using line, shape, size, quantity, value, color, texture, and locations words to describe their observations-what they see? Are they following the sentence structure?"

When you see that students are beginning to disengage, that is your cue to clap and say "pass the por-trait!"

**Step #3**: THINK: INFER/INQUIRE
Launch the investigation almost immediately and ask the class, "can YOU change the world?" We follow by showing the trailer for the movie Pay it Forward. The trailer shows a clear idea of what it means to "pay it forward."

After the clip shows, I would ask students to rephrase what it means to "pay it forward" and outline the discussion on the board.

Paying It Forward:

1. Help three people

2. Those three people help three other people

Ask students to take 2 minutes and draw a visual of this process.

"Imagine you tried this. What would you do? How much of an impact would it have. Write out three ideas you have to help others and draw a model that represents what happens if they pass it on and then the next group passes it again. How many people will you have impacted?"

After they have had a chance to reflect, Ask students to share. Have them use their mathematical model to make a social argument.

"Would paying it forward make a difference? How do you know?"

This is a discussion around what *could happen* and what students think *would happen* if they tried to pay it forward. This is not a debate with a correct or even predictable result, it is a chance to spend about 5 minutes sharing ideas around the concept of paying it forward. The more they talk about it, the more they will be ready to work on the math surrounding the concept.

**Step #4**: DO: RESEARCH (30 minutes)
Give students plenty of room in this investigation, since we want them to find a way to naturally reach an exponential model. The question asked is, "**Could paying it forward reach everyone in the world?**" Offer them two paying it forward models:

Model 1: Help 3 and have those people help 3 others (like the movie)

Model 2: Help 3 every day.

*Optional Extensions*:

Get a group of 1000 people together who will help 3 people each day. Is this a stronger model than the one in the movie. Why?

If you have a group of people willing to help 3 others each day, how large would the group need to be to reach everyone in the world in 21 days?

After you present the models and prompts, ask students, "**What do you need to solve this problem?**" Students need to know how paying it forward works (which we discussed at the start) and the current world population. They also need to have a time frame for how long it takes to complete a "good deed." This depends on what they consider acceptable as a "good deed" and what they consider to have an impact.

- Does it count if you buy someone lunch?
- Does it have to be something they can't do for themselves?
- How loosely do we define "helping someone out"?

Provide all the tools needed to solve this problem. Give them the population number (displaying the link on the projector) and have a station with graph paper, graphing calculators, etc.

Circulate and nudge students toward functions, graphs and tables, but only if they don't have another working algorithm. For example, if they really like drawing a tree to represent the growth of

paying it forward, ask them to look at a smaller population before they approach the population of the entire world. Do not discourage them from their algorithm, since the tree diagram will help them make sense of this problem in a way that is natural for them.

## CLOSURE

### RESTATE INSTRUCTIONAL OBJECTIVE AFTER LESSON

Ask students to reflect what they have discovered about modeling exponential growth and contrasting it to linear growth so far by using the "Snowstorm" strategy. The students will each write down 3 things they have learned on a half sheet of paper and then wad it up. The teacher will give a signal, in this case clap and say, "Make it SNOW" and each student will pick up a response nearest to them and read it aloud to their table mate.

Then, select a few random students to share one of the items written down on the paper they picked up. Once a few students have shared, they may begin cleaning up their area. Collect snow storm flakes and examine for misconceptions or really cool reflections to mention tomorrow.

# Day 2

| | |
|---|---|
| Introduction: Reading Portraits | 5 minutes |
| Learning Activity: Pay It Forward | 40 minutes |
| Closure: | 5 minutes |
| Total | 50 minutes |

## INTRODUCTION

### WHAT

*Today, we are going to look at the illustrations again* (point to your eyes), *think about the illustrations* (point to the top of your head), *and continue…* (show a doing gesture).

### WHY: STATE INSTRUCTIONAL OBJECTIVE

*We are going to look, think, and talk about pictures of our peacemakers to help us better understand how they changed the world based on the model exponential growth and contrast it to linear growth.*

### HOW

*These are portraits, so we need to notice the facial expression, focal point, gesture, and clothing. We do need to think about the observations you made yesterday while organizing them.*

*I am going to project the paper you received yesterday onto the board. Your job is to use your eyes to look and your brains to think how these pictures about peacemakers are the same and different. We will look and think about the pictures for one minute by ourselves and then talk as a whole group about the observations made yesterday.*

## RESTATE INSTRUCTIONAL OBJECTIVE TO TRANSITION INTO LESSON

*It is important we first look and think by ourselves so we can prepare our thoughts for the conversation about how these individuals helped spread peace around the world. What do you see is the same? What do you see that is different? Remember to support your thinking with evidence from the pictures, be prepared to answer the question "what do you see that makes you say that?"*

## LEARNING ACTIVITY

**Step #1**: OBSERVE/COMPARE/CONTRAST
Peacemaker images are placed in the middle of the circle or projected so all students can easily view.

Keep pace engaging as student first think independently (approximately one minute), then talk with a partner (approximately one minute), then facilitate a whole group discussion (approximately 2 minutes). Listen carefully to students' conversations, add to list on whiteboard of what you hear today.

Dig deeper: Ask "how did they spread their word? What do you see that makes you say that?" When students have significant observations of things that are the same and different, shift their observations to inferences. (e.g., *you notice that many of the artists used dark colors. Why do you think so many artists would use dark colors? What does the setting say about the time period?*)

Write relevant inferences/information that pertain to how these individuals spread peace around the world. It is NOT necessary to write how the images are the same/different, rather write the inferences drawn based on student observations.

**Step #2:** THINK: INFER/INQUIRE
Summarize Pay It Forward Lesson:
During their investigation from yesterday, share recorded ideas and quotes from the class overheard or written on the Snow Storm Flakes. Start off the summary by sharing some of the more compelling student ideas and use these to launch a quick conversation. For example, a student might've said, "If everyone followed through, this wouldn't take long at all. Ask the class if they agree and how they could know by showing evidence from yesterday. Students could share their approaches in tables,

graphs, functions, etc. Discuss the equation y = 3^x with questions like, what does x represent? What does y represent? How does this connect to the columns in a table and the axis in a graph?

For students who graphed the function by hand, ask students to show their work. Then, demo it on the graphing calculator and extend it by using Desmos and other online graphing calculators. Discuss the meaning of the intersection points and the reasoning as to why exponential growth is so much greater than linear growth.

Do this by simply showing a multiplicative (exponential) vs. an additive (linear) model and comparing the slopes in the linear and exponential paying it forward models.

**exponential**

3

3x3

3x3x3

**linear**

3

3+3

3+3+3

With the graphs, tables and functions shared, ask students to summarize how they can recognize an exponential relationship. They could respond in a variety of ways, but I would quote students around the following ideas:

1. Linear functions make straight lines and exponential functions make "curves."

2. Exponential functions can grow a lot faster than exponential functions.

3. Linear functions have a constant rate of change or slope. Exponential functions do not.

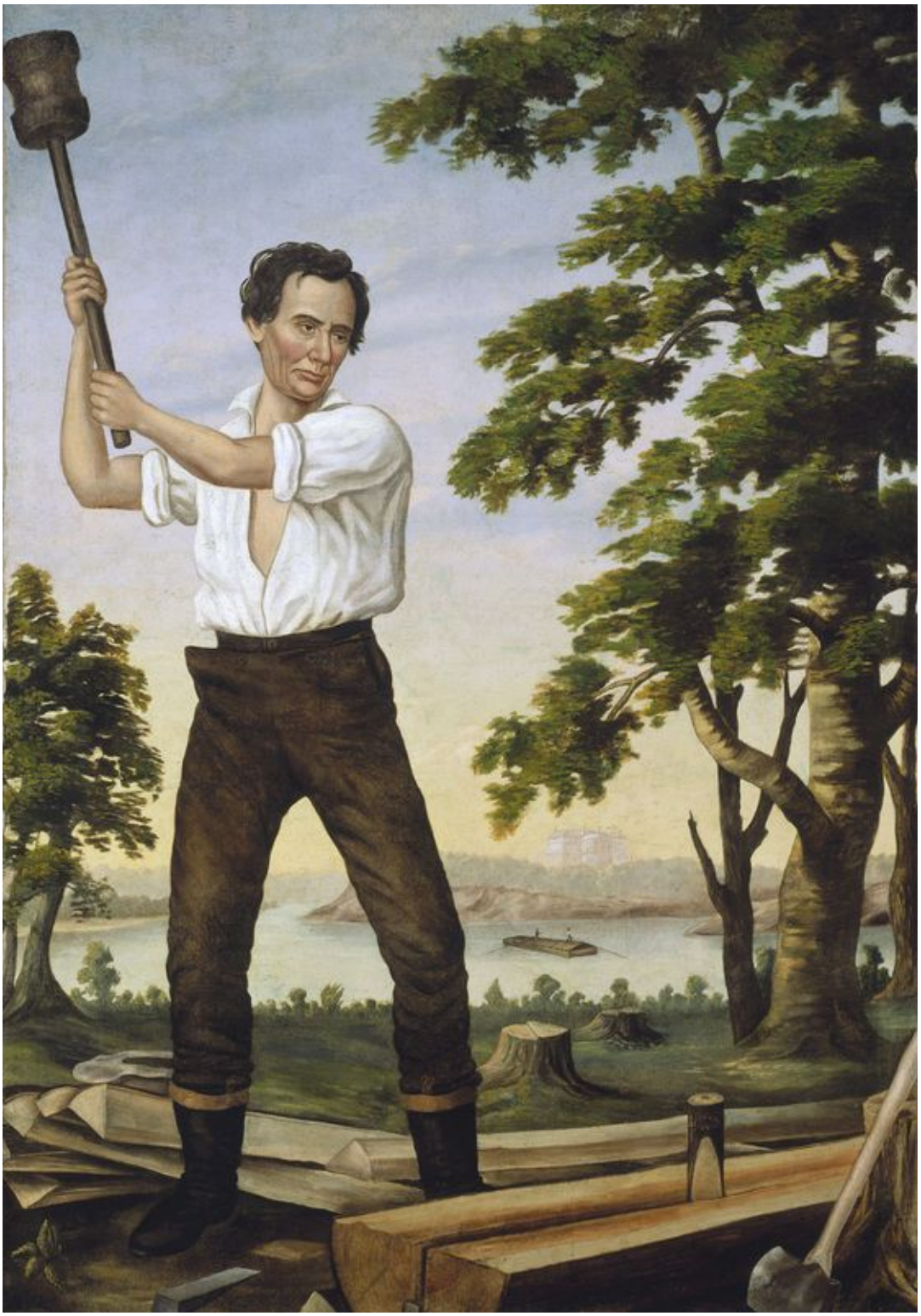
4. Linear functions look like y = mx + b and exponential functions look like y = a^x.

**Step #3**: DO: RESEARCH
Allow students to continue their research. Remind students of the learning objective: *Remember, our goal is to understand how they changed the world based on the model exponential growth and contrast it to linear growth.*

## CLOSURE

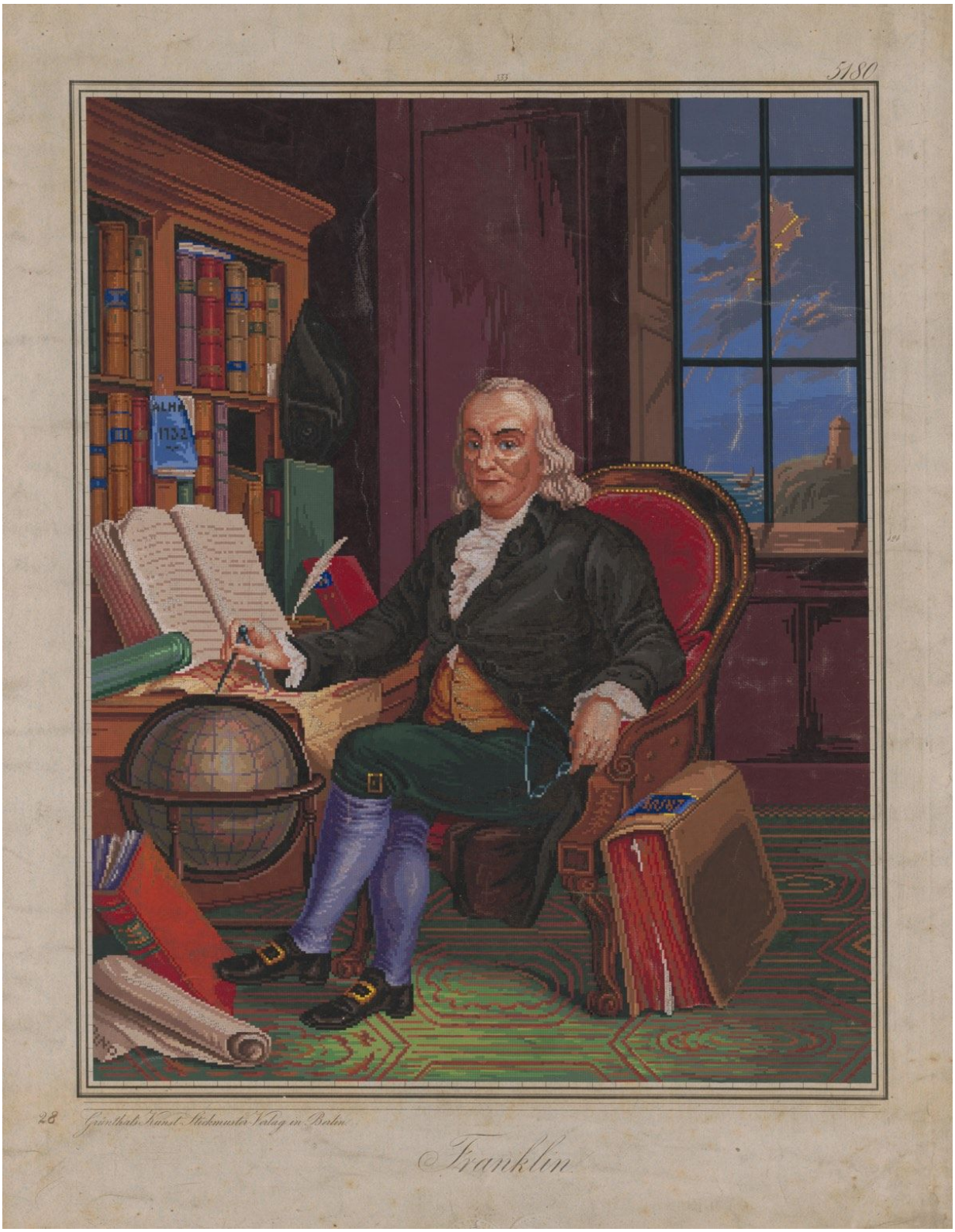
For this closing activity, students are going to participate in “Parent Hotline.” The students will e-mail their parents a sentence that begins like this: “Tonight I want to discuss what I did yesterday and today in math class. I discovered how I can change the world based on the model exponential growth and contrasted it to linear growth. I learned the model exponential growth compared to the linear

growth model (list 3 things).  In conclusion, I am going to start to pay it forward by…"  It is important the student includes teacher in the email to utilize as a formative assessment



*The Railsplitter*-1860, Unknown Artist, Oil on **canvas**

Abraham Lincoln (1809–1865) Abraham Lincoln overcame many setbacks to become the most influential American President. In his famous Gettysburg speech, he inspired the nation with his noble words and helped to bring about the abolition of slavery.

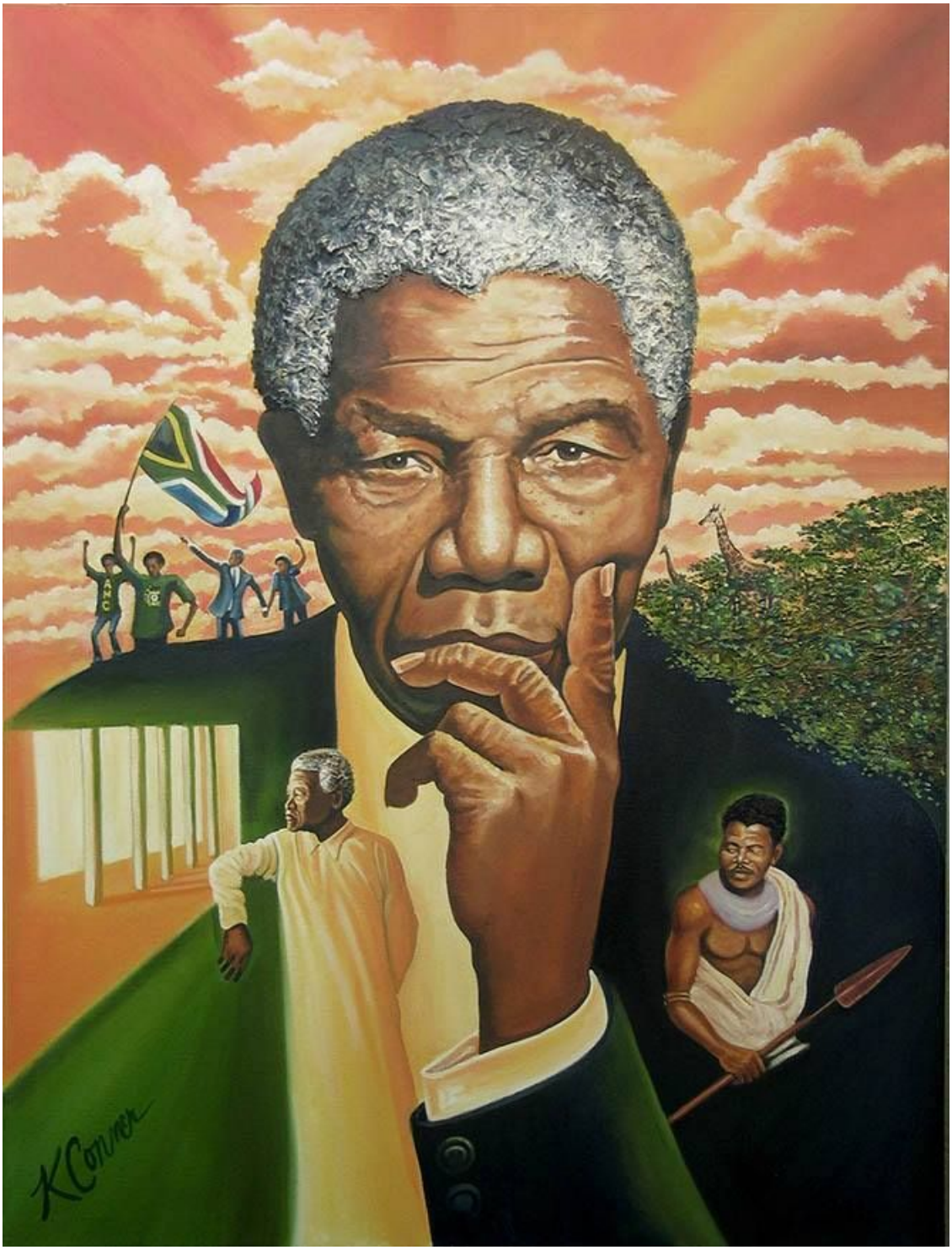



Pixelized 16-bit portrait of Ben Franklin from the 1840s

Benjamin Franklin (1706–1790) Great polymath and promoter of American ideals at home and for the country. A practical man of great dynamism and good character.

Saatchi Online Artist: Konstantin Altunin; 2011, "Restructuring. Democracy. Publicity. Mikhail Gorbachev"

Mikhail Gorbachev (1931– ) Had the courage, tenacity and strength of character to give up the absolute power of Soviet Communism. Moved the Soviet Union to democracy and respect for human rights. Gorbachev enabled the Berlin Wall to come down giving freedom to Eastern Europe.

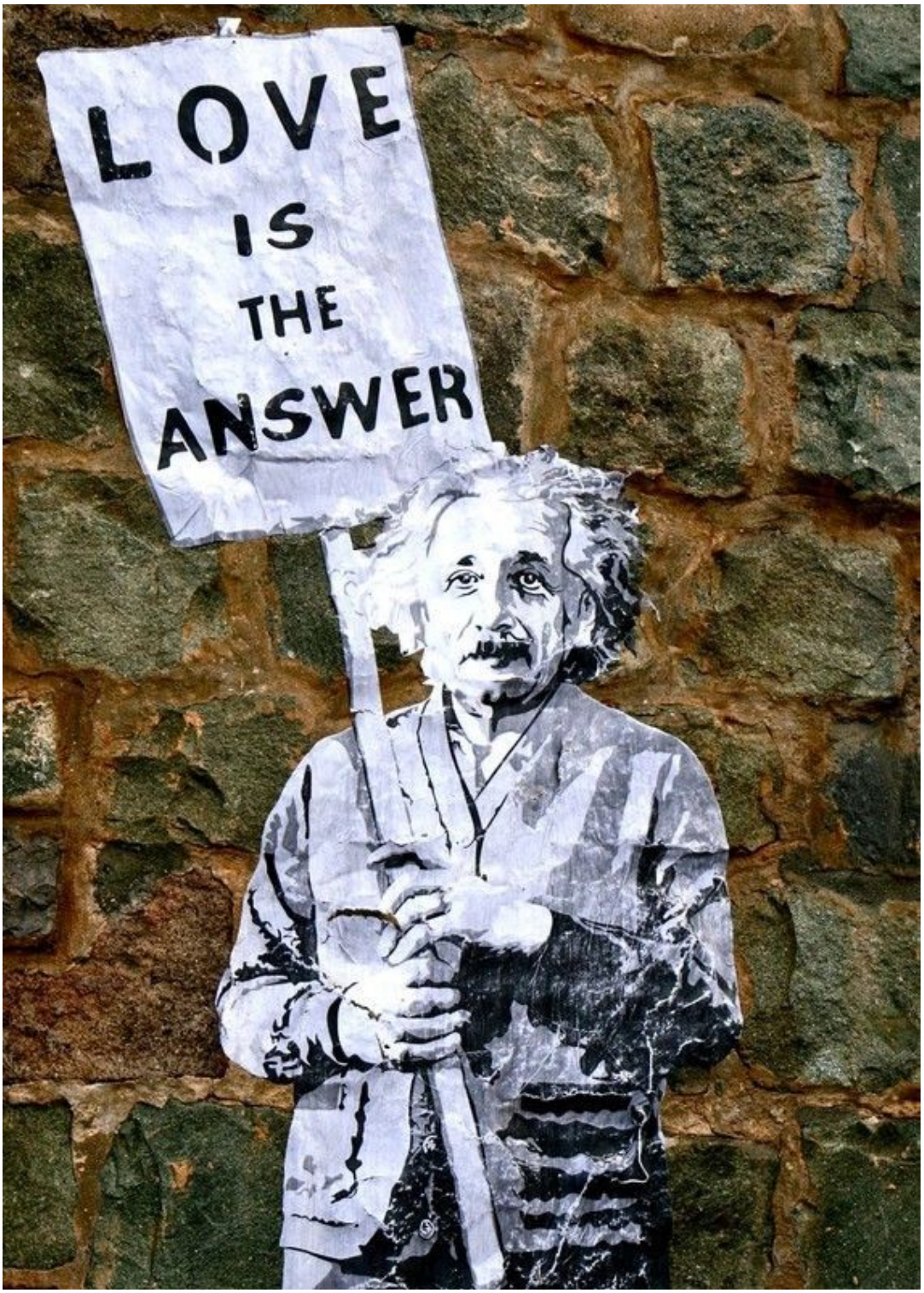



by Keith Conner

Nelson Mandela (1918–2013) Campaigned for justice and freedom in his country South Africa. Mandela, spent 27 years in jail for his opposition to apartheid. On release, he healed the wounds of apartheid by his magnanimous attitude to his former political enemies.

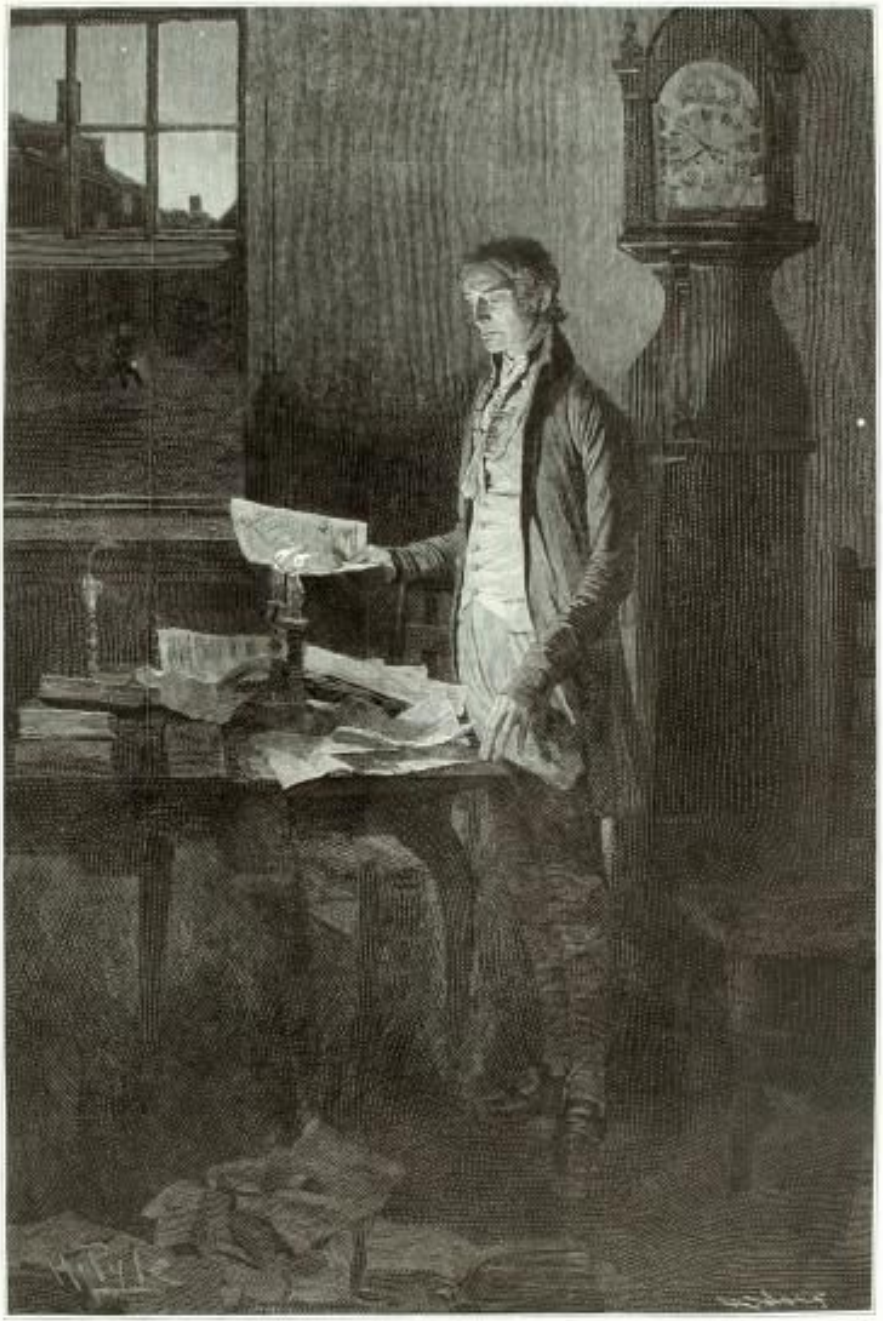

*Sermon on the Mount* - Henrik Olrik Canvas

Jesus Christ (c.5BC–30AD) Spiritual Teacher and inspiration of Christianity. Taught a message of love, forgiveness and faith. Born in a turbulent period of Roman rule, after his crucifixion, his message inspired millions around the world.



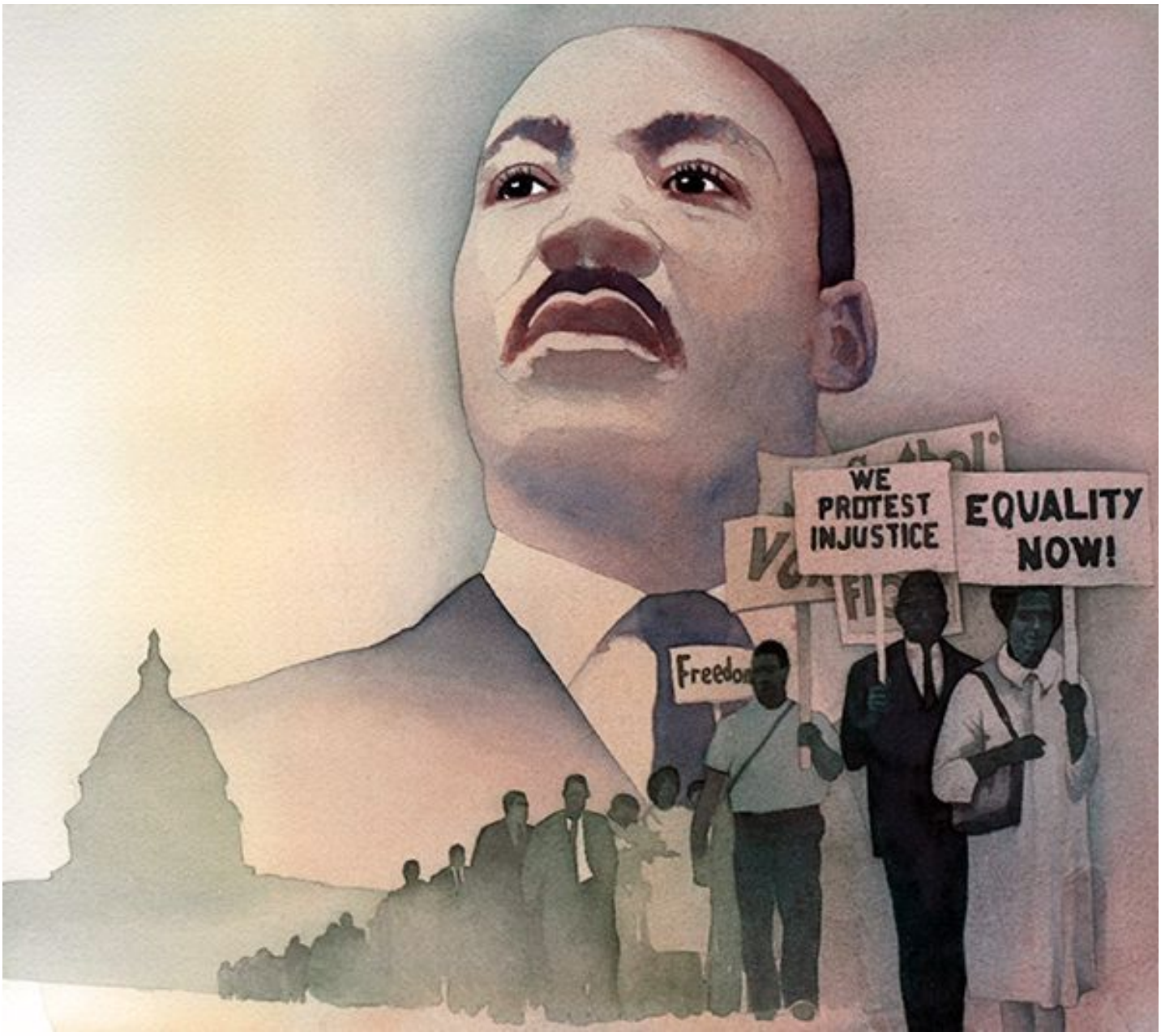

*Love is the answer. El amor es la respuesta*

Albert Einstein (1879–1955) Einstein's theories of relativity were a very significant scientific breakthrough. As well as being a genius scientist, Einstein was also a champion of human rights and campaigned for a more peaceful world.



Henry Wolf, *Thomas Jefferson, Writing the Declaration of Independence*, 1898, photomechanical wood engraving on paper,

Thomas Jefferson (1743–1826) One of America's founding fathers, Jefferson helped draft the Declaration of Independence and fostered a belief in human rights. Amongst other achievements, Jefferson passed one of the first bills on religious tolerance in his state of Virginia.




*A Land of Big Dreamers*, a watercolor created by Neil Walman

Martin Luther King (1929–1968) Inspiring leader of the non-violent civil rights movement. Inspired millions of people, black and white, to aspire for a more equal society.


Harriet Tubman, pictured between 1860 and 1875. The woman who will soon become the first African-American to grace an American currency note self-funded many of her heroic raids to save slaves by cooking.

*Leonardo da Vinci* (self-portrait), c. 1478/79. (Courtesy of Peter Ackermann)

Leonardo da Vinci (1452–1519) One of the greatest minds in human history. There were few areas of art and science that Leonardo didn't delve into. In many areas, he was a couple of centuries ahead of scientific discovery. Da Vinci helped make great advances in anatomy, astronomy, physics, science and others.

Description: *Portrait of Helen Keller* as an adolescent reading a large embossed book that is resting in her lap. Date: circa 1894 Acquisition Note: from the Nella Braddy Henney collection Format: black and white photograph

Helen Keller (1880–1968) Despite her condition of both deafness and blindness, she learned to read and write, becoming a champion of social issues and helping to improve the welfare of deaf people.

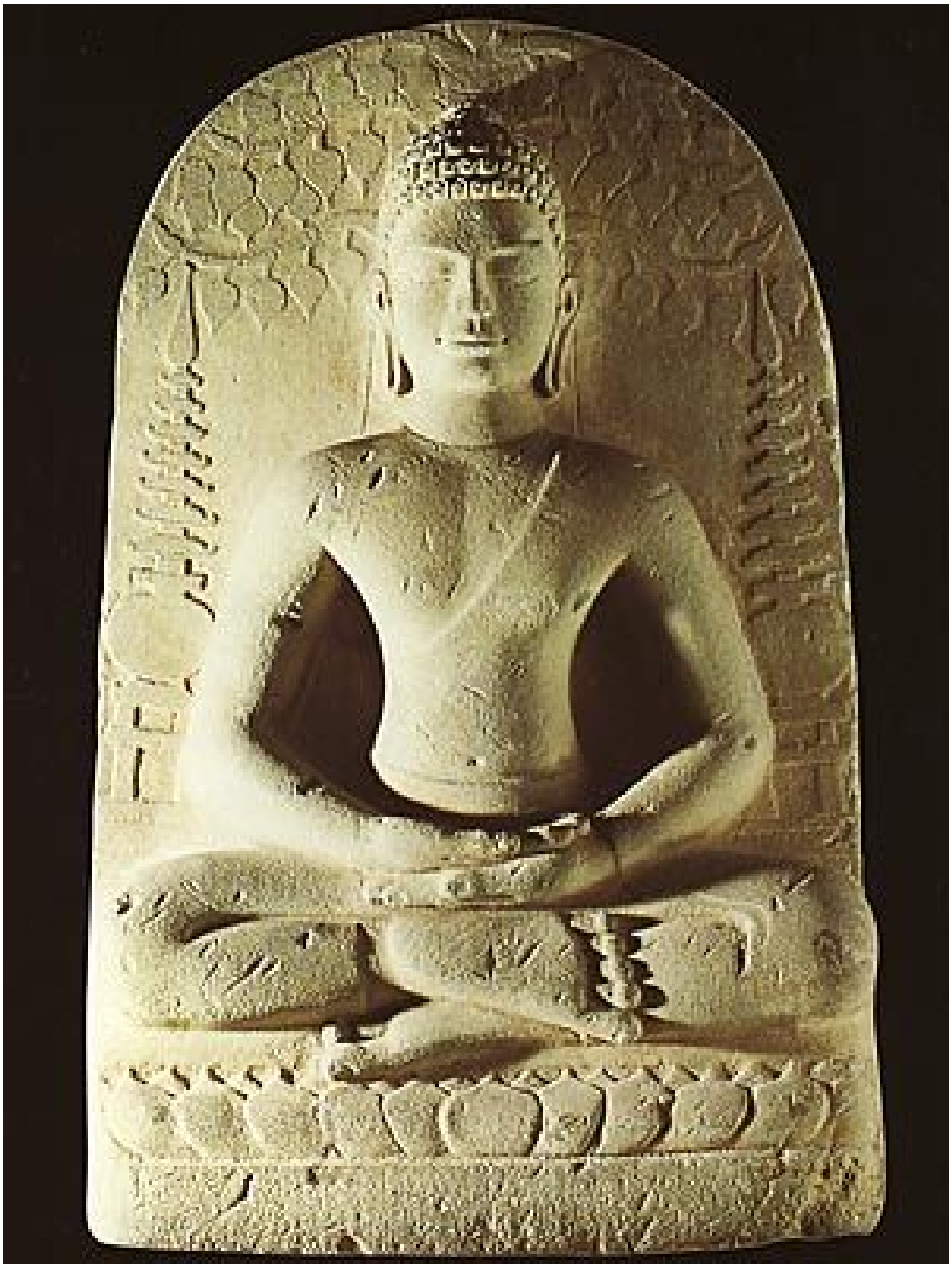
Mother Teresa by Arabinda Aich

Mother Teresa (1910–1997) Lived a life of poverty to try and ameliorate the conditions of others. Her devotion and compassion inspired the lives of many thousands she came into contact with.

*Mahatma Gandhi spinning*

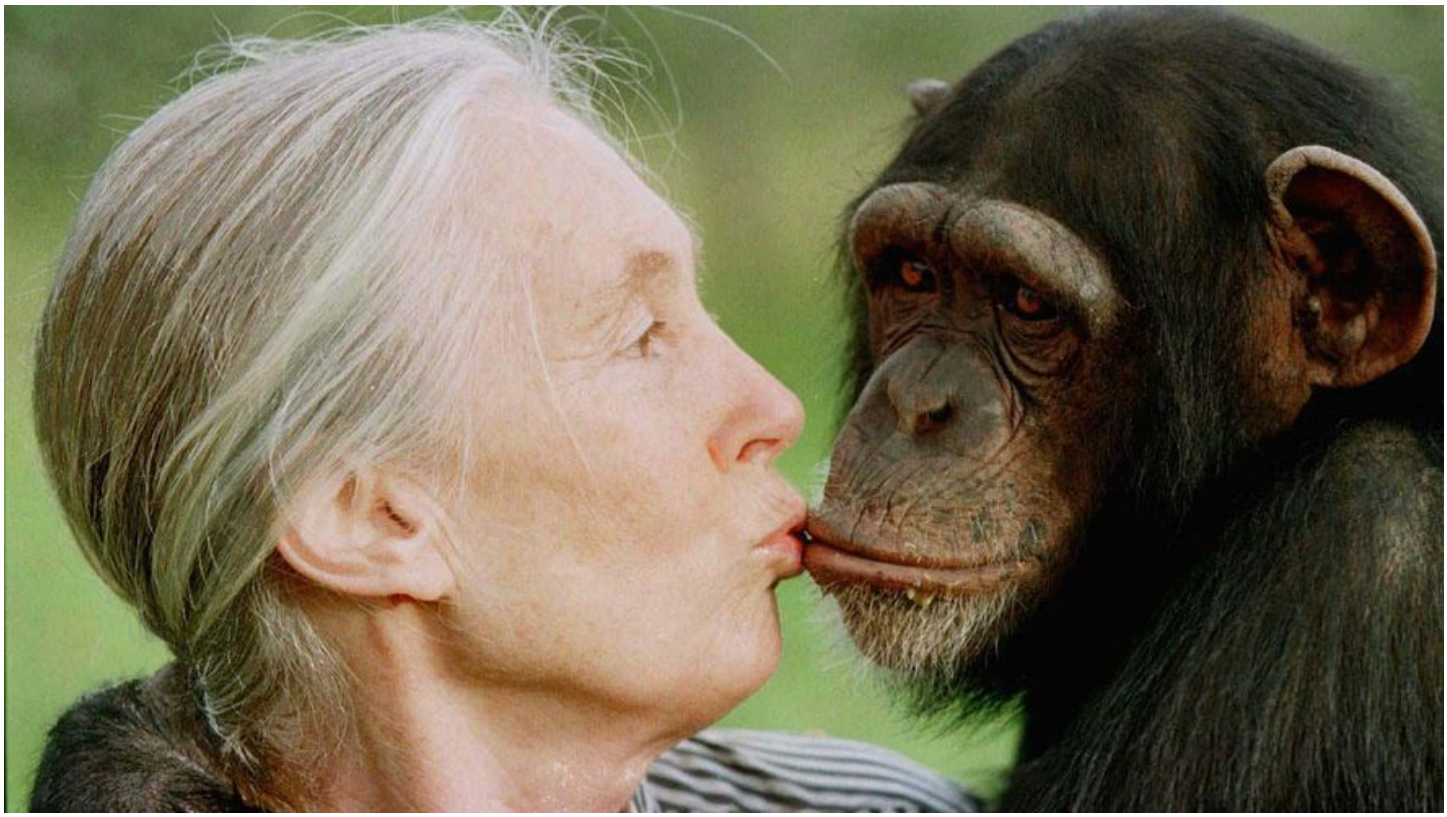
Medium: Oil on canvas

Mahatma Gandhi (1869–1948) Gandhi was the principle figurehead of the Indian independence movement. Gandhi followed a philosophy of nonviolence and peaceful protest; he also sought to improve conditions for women and people disenfranchised by the caste system.



Dvaravati art - Sitting Buddha in Meditation Posture.  8 - 10th century A.D.

Buddha (c 563 BCE–483 BCE) The Buddha was a young prince who gave up the comforts of palace life to seek the meaning of life meditating in the forest. After gaining realisation, the Buddha spent the remainder of his life travelling around India teaching a middle path of meditation and inner peace.



Jane Goodall gives a little kiss to Tess, a female chimpanzee at the Sweetwaters Chimpanzee Sanctuary near Nanyuki, 110 miles north of Nairobi, Kenya, on December 6, 1997. Photo by: Jean-Marc Bouju

Jane Goodall (1934– ) Groundbreaking researcher into the behaviour of chimpanzees. Goodall became a noted campaigner and activist for environmental protection and kindness to animals.